3D Mapping Aided GNSS Using Gauss-Newton Algorithm: An example on GNSS shadow matching

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ABSTRACT

Global navigation satellite system (GNSS) provides an all-time positioning service that covers the entire world. Technology development impacts our daily life, positioning becomes essential. At meanwhile, urbanization induced the side-effect to position, especially for GNSS. High rise buildings and obstacles can block and reflect the GNSS signals. Using a 3D building model is a new era to aid urban positioning, namely 3D mapping aided (3DMA) GNSS. Conventional 3DMA GNSS uses position hypothesis candidates to estimate the receiver location. Throughout simulation at each distributed candidate, the one with the highest similarity to measurements tends to be the receiver location. Two obvious shortcomings of positioning hypothesis candidate are: first, candidate distribution must cover the truth location to provide satisfactory performance. Second, the computation load is proportional to the number of distributed candidates, and unwanted computation load may be caused during the estimation process. This study tries to overcome these limitations by replacing the positioning hypothesis candidates with nonlinear least squares to estimate the receiver location. Contributions of this study will be reducing the computation load requirement while maintaining the identical positioning performance. We selected shadow matching as the showcase to demonstrate the impacts of our proposed method on the 3DMA GNSS with actual GNSS data from a low-cost receiver.

INTRODUCTION

Rapid urbanization introduced a significant uncertainty for the GNSS positioning. That is the reason why we usually cannot achieve a satisfactory positioning performance. The high-rise buildings and obstacles can block and reflect the GNSS signals. This introduced two main phenomena, which are non-line-of-sight (NLOS) reception and multipath effect [1], and limited the positioning performance in the urban environment [2]. Different studies have been proposed to mitigate or eliminate these negative effects.

Beneficial by the convenience of 3D model retrieval, such as combining the satellite images and airborne LiDAR, where the former and latter provide 2D building contour and building height, respectively [3]. A complete review of the making of large-scale 3D building models can be found at [4]. More 3D building model resources are available for commercial use. This explores a new era on using the 3D building model to improve the positioning accuracy in the urban, namely 3D mapping aided (3DMA) GNSS [5]. 3D building model provides two features for GNSS positioning, satellite visibility prediction and signals transmission path propagation. Further introduce several popular 3DMA GNSS algorithms, shadow matching [6, 7], ray-tracing GNSS [8, 9], likelihood-based ranging [10], and skymask 3DMA [11]. A previous study has shown that 3DMA GNSS can improve position accuracy by about 25% in an urban environment [10].

Conventional 3DMA GNSS uses a positioning hypothesis candidate-based approach to estimate the receiver location. Based on the simulation of each distributed candidate, we can evaluate the similarity of prediction and measurement. Higher similarity means a larger likelihood for the candidate to be the receiver location. This candidate-based approach comes up with two main limitations to provide the best performance. First, a reliable initial position is required for the candidate distribution. To be more precise, candidates must cover the ground truth to ensure it is being examined to get the best performance. Otherwise, the sampling area needs to be enlarged for candidate distribution. And this causes the second limitation on bringing extra computation load to the receiver and becomes difficult for practical implementation. The evaluation process only wants to find out the candidates near the receiver location. Other low similarity candidates are actually not our goal during the evaluation process.

Up to the discussion here, we want to estimate the position effectively while maintaining accuracy. This study is going to replace the positioning hypothesis candidate-based approach with a nonlinear least-squares approach for snapshot solution. We are going to use shadow matching as a showcase in this study. With the actual GNSS data recorded in Hong Kong, the nonlinear least-squares approach can provide a similar positioning accuracy than the positioning hypothesis candidate-based approach, but with higher estimation efficiency by reducing the sampling positions.

OVERVIEW OF THE PROPOSED ALGORITHM

3DMA GNSS is a hot topic for GNSS society, and more product integrates the 3DMA GNSS to provide an acceptable positioning in the urban environment. The existing algorithm uses the positioning hypothesis candidate-based approach to estimate the position solution. The procedure of the positioning hypothesis approach is first distributing the positioning candidates around the initial guess position. We estimate the similarity of the features between measured and predicted on each candidate's location — the higher similarity, the higher possibility of the receiver location. The matching feature to be used depends on the 3DMA GNSS algorithms. Using shadow matching as an example, it uses satellite geometry and visibility to estimate the position. The actual GNSS signal reception matching with the visibility prediction by the 3D model becomes the key of shadow matching to resolve the receiver location.

To summarize 3DMA GNSS in one sentence, 3DMA GNSS is to find a location that best matches the 3D building model prediction and receiver measurements. Thus, we can mathematically express this statement,

$$\mathbf{x} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{y} - \hat{\mathbf{y}}\| = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{F}(\mathbf{x})\| \tag{1}$$

where $\mathbf{x} = [E, N]$ is the state, also known as the east E and north N position in our problem. $\mathbf{y}$ is the actual observation from the receiver. $\hat{\mathbf{y}} = \mathbf{G}(\mathbf{x})$ is the estimated value using the 3D building model. Both are column vectors with the length of the number of the available satellite. And we can foresee that the 3DMA GNSS is an optimization problem, as long as we can express the physical model of different algorithms.

This study uses shadow matching as the demonstration of adopting the nonlinear least squares to resolve the solution. We revisit the state-of-art shadow matching proposed in [10], the visibility consistency, P_j^i, for i-th satellite at j-th candidate is given by,

$$P_j^i = P_{C/N_0}^i P_{BB,j}^i + \left(1 - P_{C/N_0}^i\right)\left(1 - P_{BB,j}^i\right) \tag{2}$$

where P^i_{C/N_0} is the probability that the received signal is direct LOS one. It is determined from the receiver's C/N_0 measurements. $P^i_{BB,j}$ is the probability that the satellite is predicted to be direct LOS. And the prediction is based on the building boundaries precomputed from the 3D building model.

In which we wish to find a position that can maximize the objective function, P^i_j in (2). As a result, expressing the function P^i_j with position state $\mathbf{x}$ is important. After combining with (1), that yields to,

$$\mathbf{x} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{G}(\mathbf{x})\|^2 = \underset{\mathbf{x}}{\operatorname{argmin}} \|-\log(\mathbf{P_x})\|^2 \tag{3}$$

where $\mathbf{P_x} = [P^1_{\mathbf{x}} \ldots P^i_{\mathbf{x}}]^{\boldsymbol{T}}$ is a column vector with the number of available satellites related to the state, or known as positioning in this situation. Taking $-\log(\cdot)$ in (3) is to minimize our objective function. Look into function $\mathbf{P_x}$, the only term that related to the state $\mathbf{x}$ is the model visibility prediction, P^i_{BB}. We can express the probability of a satellite being LOS at a position,

$$P^i_{BB}(\mathbf{x}) = \tanh\left(\alpha \times \left(EL^i - BB_j\left(h^{AZ^i}_{\mathbf{x}}, d^{AZ^i}_{\mathbf{x}} | \mathbf{x}\right)\right)\right) \times \tau + \frac{1}{2} \tag{4}$$

$$BB_j\left(h^{AZ^i}_{\mathbf{x}}, d^{AZ^i}_{\mathbf{x}} | \mathbf{x}\right) = \tan^{-1} \frac{h^{AZ^i}_{\mathbf{x}}}{\sqrt{\left(E - d^{AZ^i}_{\mathbf{x}} \sin(AZ^i)\right)^2 + \left(N - d^{AZ^i}_{\mathbf{x}} \cos(AZ^i)\right)^2}} \tag{5}$$

where AZ^i, EL^i represent the azimuth and elevation angle of the the i-th satellite, respectively. $h^{AZ^i}_{\mathbf{x}}, d^{AZ^i}_{\mathbf{x}}$ are the highest building height and horizontal distance at AZ^i with respect to position $\mathbf{x}$, and these are obtained from the pre-generated skymask. And function $BB_j\left(h^{AZ^i}_{\mathbf{x}}, d^{AZ^i}_{\mathbf{x}} | \mathbf{x}\right)$ is actually calculating the skymask elevation angle at AZ^i, the variables E, N are the state $\mathbf{x}$ that we wish to optimize. The denominator here decomposes the horizontal distance to the building, adds the change of state, then reforms the total distance. α is the tuning factor for the classification to make the building boundary classification blunt. $\tau = \frac{1}{2}(\tau_{\max} + \tau_{\min})$ is the scaling factor. It scales the output of $P^i_{BB}(\mathbf{x})$ falls between the range $[\tau_{\min}, \tau_{\max}]$.

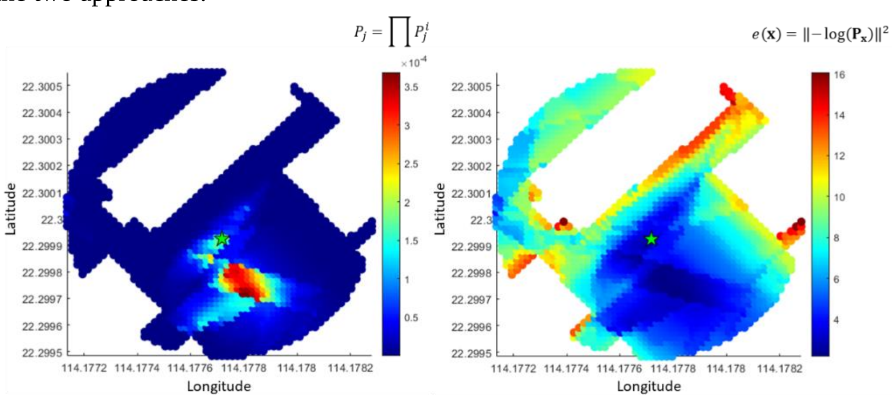
After we obtain the model that expresses by the state $\mathbf{x}$, we can optimize it through the nonlinear least-squares approach. There are three most popular nonlinear least-squares approaches, gradient descent, Gauss-Newton, and Levenberg-Marquardt method. Where all three methods optimize our objective function iteratively. And once the state is updated, the position-dependent parameter, such as $h^{AZ^i}_{\mathbf{x}}$ and $d^{AZ^i}_{\mathbf{x}}$ are also updated accordingly to provide more accurate parameters during optimization. The state optimization and update at k-th epoch can be done by the Gauss-Newton method [12],

$$\mathbf{x}^{k+1} = \mathbf{x}^k - [\mathbf{J}(\mathbf{x})^{\mathbf{T}}\mathbf{J}(\mathbf{x})]^{-\mathbf{1}}\mathbf{J}(\mathbf{x})^{\mathbf{T}}\mathbf{G}(\mathbf{x}) \tag{6}$$

where $\mathbf{J}(\mathbf{x})$ is the jacobian matrix and it is calculated by $\boldsymbol{\nabla}\mathbf{G}(\mathbf{x})$.

In this study, we only optimize the position solution with the Gauss-Newton method. As Gauss-Newton is the only method that uses the residual of the objective function to determine the converging speed. Thus, there is not necessary to consider the strategy to tune the parameters through the optimization process.

This study proposed a new approach to model the prediction function for shadow matching. Therefore, it is necessary to compare the objective function output on the candidate-based shadow matching and the proposed method by plotting their heatmap. Figure 1 shows the heatmap for the two approaches.



(a) (b)

Figure 1: Heatmap on (a) candidate-based shadow matching; and (b) proposed method, modelling the prediction function with position state $\mathbf{x}$ in (2)-(5). Noted that variable $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$ is given by on each position $\mathbf{x}$. The green star is the ground truth.

Noted that the candidate-based shadow matching in Figure 1(a) targets maximizing the function, so the red part is what we want in this problem. While the proposed modelling in Figure 1(b) tries to minimize the function. Therefore, the blue part is the objective in this case. As a result, two heatmaps are inversely proportional to each other. And we can see that the same trending can be observed for both heatmaps, which means that the same performance should be obtained during the evaluation for the position.

At meanwhile, the prediction function in (4) and (5), there are two position-dependent variables, $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$. These two variables are important during optimization as they describe the surrounded building height and horizontal distance. They should change whenever the estimated position state $\mathbf{x}$ changed. And we also compare the heatmap for two cases, when the variables are given by initial location that far and close to ground truth, respectively, as shown in Figure 2.

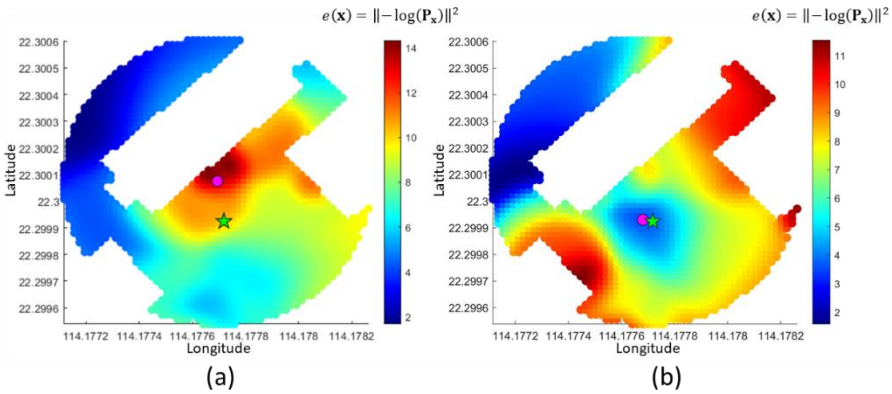


Figure 2: Heatmap on proposed modelling with position state $\mathbf{x}$ dependent variable $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$ based on the initial position (purple point), where initial at location (a) far from ground truth; and (b) close to ground truth. The green star is the ground truth.

From Figure 2(a) heatmap, we can observe that when the initial location is far from the ground truth, which means the $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$ are probably different from the actual location. As a result, the minimized location is wrong in this case. While the initial location in Figure 2(b) is close to the ground truth, so the variables should be the same as the ground truth. And we can observe that the minimized location is near the ground truth. This means that the solution should be able to converge to the correct location.

EXPERIMENTS RESULTS AND ANALYSIS

This study is going to demonstrate the proposed approach with actual recorded data in Hong Kong. The result is divided into two main parts, first is the positioning accuracy, and another part is the required computation load for two methods. The positioning result in the map plot is shown in Figure 3, and the statistic is presented in Table 1, where position error is categorized in root mean squared (RMS), mean, standard deviation (STD), maximum (MAX), minimum (MIN) positioning error.

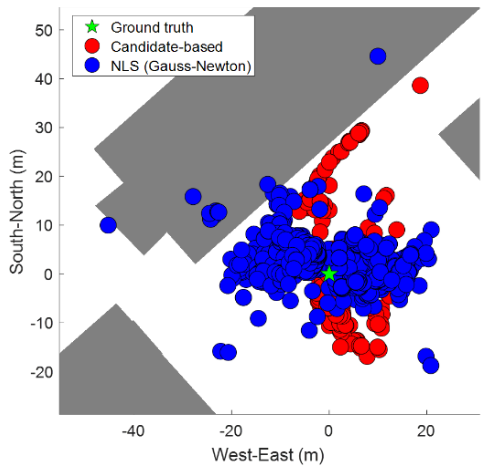


Figure 3: Positioning result of the smartphone data in 2D map view.

Table 1: Positioning error statistics of the smartphone data.

	Candidate-based shadow matching	Nonlinear least squares approach (proposed method)
RMS	8.26	11.11
Mean	6.78	9.74
STD	4.73	5.33
MAX	42.90	46.39
MIN	0.31	1.32

The result shows that the proposed method can provide similar performance in terms of positioning accuracy. The difference between the candidate-based and nonlinear least squares approach is the number of positions or candidates that must be sampled. Nonlinear least-squares uses mathematics calculation to reduce the number of sample candidates. As a result, the nonlinear least-squares approach should be able to optimize the estimation process. As a result, we are going to estimate the computation load of two methods. The main difference between the two approaches is the number of trials required to sample, so we mainly compare this. The computation load for the two methods is shown in Figure 4. The y axis on the left-hand side in red represents the number of candidates for the candidate-based approach. The right-hand side in blue represents the number of the estimated position for the proposed method. Noted that the number of satellites is identical for both methods, so the figure only shows the number of candidates or sampled positions.

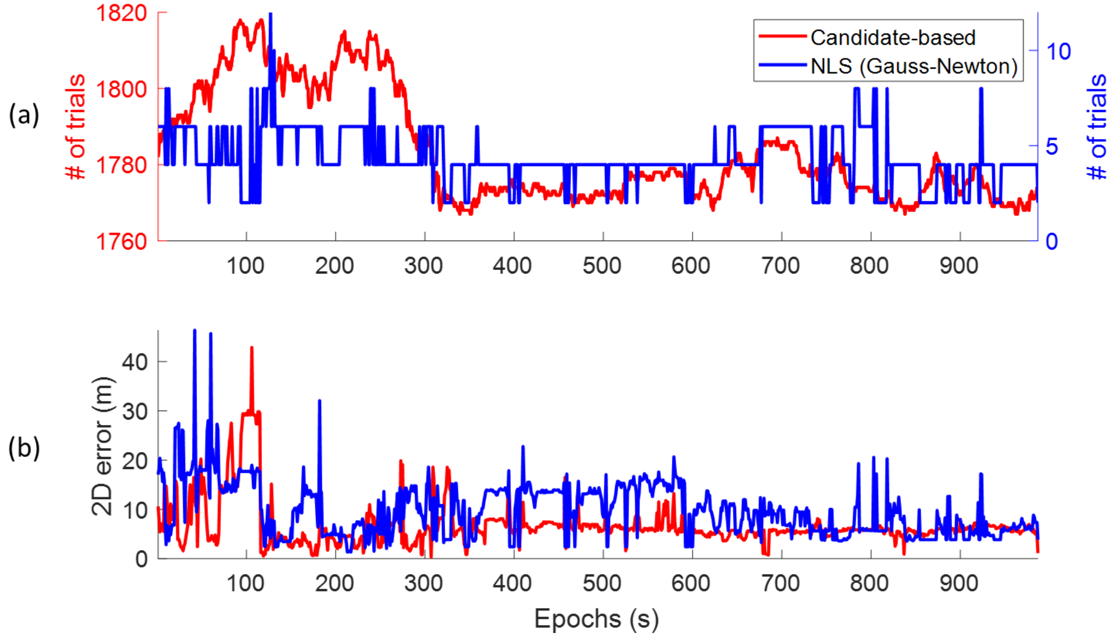


Figure 4: Positioning result on (a) number of sampled positions for candidate-based (red) and nonlinear least-squares (blue); (b) 2D error.

The sampling area is set to a 40m radius with candidates separation of 2m for a candidate-based approach. The require sampling candidate is more than 1500 through the whole experiment. After replacing the position estimation with Gauss-Newton, the number of estimations was reduced to less than 12 throughout the experiment. A larger positioning error can be found in this experiment result. The reason could be the proposed prediction function cannot perfectly model the actual environment. Although the location-dependent variables, $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$, are updating at each iteration, the surrounding building feature is still considering a single location but not the whole environment. As a result, the prediction function should also consider to update $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$ when estimating the visibility. In other words, variables $h_{\mathbf{x}}^{AZ^i}$ and $d_{\mathbf{x}}^{AZ^i}$ should be a E and N related function to better estimating the building information.

CONCLUSIONS AND FUTURE WORK

This study presented a novel way to improve the efficiency of position estimation for 3DMA GNSS. A snapshot-based nonlinear least squares approach is proposed to replace the candidate-based approach. Two main contributions were made in this study, first, expressed the 3DMA GNSS in a mathematical way as the objective function. And the second contribution is to optimize the objective function as the least-squares problem. This can reduce the number of sampling positions to increase efficiency. The actual recorded data have shown that the proposed method can provide a similar positioning accuracy but a much lower sampling position number through the estimation process. This study only used shadow matching as the demonstration, the potential of the proposed method can still be extended in several domains.

However, the proposed modelling is imperfect for practical usage. Where the prediction function includes the location-dependent variables, this limits the performance of the algorithm. When the initial location consists of a large error and wrong variables are given, the solution may go in the wrong direction. Furthermore, the algorithm cannot determine the correct converging direction when there is not enough feature for the estimation (e.g., worst satellite distribution).

Implementation wise, the result shown in this study is only a prototype. There are different opensource optimization libraries, such as Ceres Solver, GTSAM, and GraphGNSSLib, that can further optimize the performance of the proposed method. Where we can easily tune the parameters through the optimization process, such as the damping factor in the Levenberg-Marquardt method.

Furthermore, this study only employs the Gauss-Newton algorithm as the showcase. A complete study is required on adopting different nonlinear least-squares, such as gradient descent and the Levenberg-Marquardt method. This lets us better understand the adaptation of each algorithm to 3DMA GNSS.

In terms of the 3DMA GNSS generalization, other ranging-based 3DMA GNSS algorithms will model as the mathematical problem and estimate the position as a batch. Besides the benefits of increasing the efficiency, solving 3DMA GNSS as a least-squares problem can easily plug and play into different graph problems that are great potential to integrate with a bunch of algorithms. Once we can express the 3DMA GNSS as an objective function with the position state, we can consider the function as a node inside the graph problem.

In the time domain, we can correlate the snapshot least-squares problem with time correlation, such as incorporating Doppler measurements. As a result, we can solve the graph problem as a batch to further optimize the performance of 3DMA GNSS, which is the concept of factor graph optimization (FGO).

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